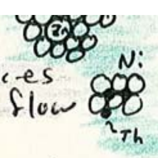
Ideal Shear Strength derivation

$\overset{a}{-}$ a≥b Comparable size

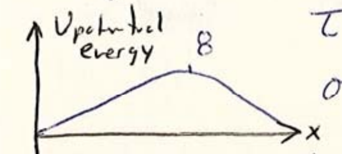
Slip planes plastic flow



$\tau \propto \frac{dU}{dx}$

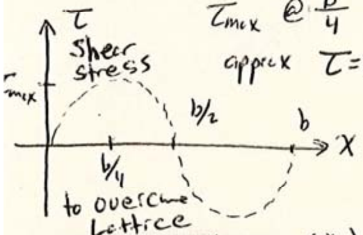
$\sigma \propto \frac{dU}{dx}$

Crystal energy max when Horizontal 1/2 displacement.



τ_{max} @ $\frac{b}{4}$ $\frac{x}{b} << 1$

approx $\tau = \tau_i \sin\left(\frac{2\pi x}{b}\right)$



$\tau = G\gamma$
$= G\left(\frac{x}{a}\right)$

$\tau = \tau_i\left(2\pi\frac{x}{b}\right) = G\left(\frac{x}{a}\right)$

$\rightarrow \boxed{\tau_{i_{max}} = G/2\pi}$

τ_i metal 10^{+3} τ_{ideal}. measured value smaller account by dislocation

Austenite, C γ-Fe FCC
Ferrite, C α-Fe BCC
phase diagram

Stress equilibrium required to overcome lattice resistance

only (rods) martensite captured via quenching

Ferrite Cementite lowalloy steel

$\gamma = \frac{x}{a}$ Strain

$\frac{d\tau}{dx} = \frac{2\pi\tau_{max}}{b}$ stress theoretical shear strength.

$\gamma = \frac{x}{a}$ δFe Bcc γFe Fcc αFe Bcc

Properties of dislocations

3/2

f = Friction per unit length.

$$\underbrace{[\tau \cdot l_1 \cdot l_2]}_{Force}\ b = (f \cdot l_1) \cdot l_2$$

Burger vector ↑ length dislocat ↑ distance travel ↑

$f = \tau \cdot b$ for all dislocations.

$\tau_y = \frac{f}{b}$ $\frac{\text{Frictional force}}{\text{Burgers factor}}$

Line tension

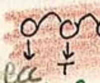
Yield stress dislocation poly $\sigma_y = 3\tau_y$

Energy per unit length $\frac{\pi G b^2}{8} \simeq T \approx \frac{Gb^2}{2}$

$\tau_y = \frac{f_i}{b} + \frac{f_{ss}C^{1/2}}{b} + \frac{f_o \sim \frac{2T}{L}}{b} + \frac{f_{wh}}{b}$

Bond Strength ↗ dislocation density ↗ density ↗ precip ↗ Work hardening dislocation density

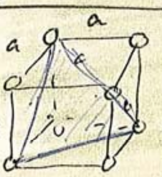
$C\uparrow \tau_y\uparrow$ $\uparrow f_{ss}$ $f_o\uparrow$ $\tau_y\uparrow$



$2T = \tau_y b l$

F.CC. metals

Cu, Al, Pt, A Stainless st



Shearing plane $\{1,1,1\}$

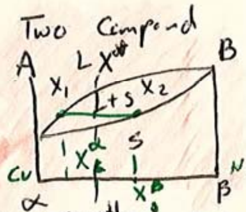
mag $\frac{a}{\sqrt{2}}$ Direction $\{1,1,$

dislocations $n = \frac{\alpha\ \tau\ d^{size}}{G\ burger}$ const

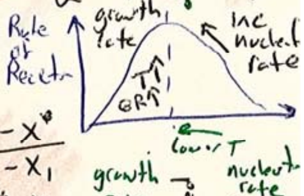
Hall petch $\left(\frac{f_{GB} \cdot G}{}\right)^{1/2}$
$= \tau_y = \beta d$

Grain Boundary ↑ Yield Stress.

Two Compound



Fraction of L $L = \frac{X_S^\beta - X_o}{X_S^\beta - X_L^{Ni}}$



X_1 solid % $\frac{X_2 - X^*}{X_2 - X_1}$

X_2 liquid % $\frac{X_B^* - X_1}{X_2 - X_1}$

growth rate · Reaction Rate → nucleation rate → Reaction Rate

Poly crystal (dislocation) $\sigma_y = 3\tau_y$

Cracks across grain → transgranular fracture

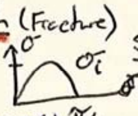
Brittle Fracture → intergranular fracture → Cracks along grain Boundaries

F.S Fracture Surface
· fibrous linkage F.S. Rough
· Void formation + Linkage F.S. Rough

Ductile fracture

Cleavage Strength (Fracture)

Separation of particles



Consider $\sigma = \sigma_i \sin\left(\frac{2\pi\delta}{\lambda}\right)$

Small stress sm

$\sigma = E\varepsilon = E\left(\frac{\delta}{a_0}\right)$

$E\left(\frac{\delta}{a_0}\right) = \sigma_i\left(\frac{2\pi\delta}{\lambda}\right)$

$\sigma_i = \frac{E}{\pi}\left(\frac{\lambda/2}{a_0}\right)$

work to separate adjacent planes. $W = \int \sigma\, d\delta$

used to create two new surfaces. with surface energy Γ_s

$W = \sigma_i \cdot \frac{\lambda}{\pi} \left[\frac{J}{m^2}\right]$

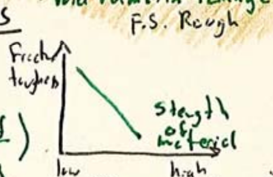
$0.1\ J/m^2 - 1\ J/m^2$

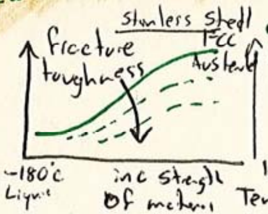
$\lambda = \frac{2\pi\Gamma_s}{\sigma_i} \Leftrightarrow \sigma_i = \frac{E}{\pi}\left(\frac{\lambda/2}{a_0}\right)$ $\sigma_i = \sqrt{\frac{E\Gamma_s}{a_0}}$

$\sigma_i \approx \left(E/10 \sim \frac{E}{3}\right)$ existing cracks.

↓ Scale ↓ flaw size ↑ fracture strength

Stress

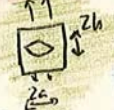




Charpy Impact energy

stress concentrator

$\sigma_{22}(\text{local}) = K_t\sigma^\infty = 3\sigma^\infty$



$\sigma_{22}(\text{local}) = \left(1 + 2\frac{a}{b}\right)\sigma$

Sharp crack $\sigma_{22}(\text{local}) = 2\frac{a}{b}\sigma^\infty$

$\rho = b^2/a$

$\sigma_{22}(\text{local}) = 2\sqrt{\frac{a}{\rho}}\,\sigma^\infty$

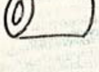
Rate of bond Formation

thermally activated process. $\exp\left(\frac{-E_a}{kt}\right)$

Carbon steel BCC Ferrite

Against Yield & Fracture Crack tip

$P_{fracture} = \frac{K_{IC}}{Q\sqrt{\pi a}} \cdot \frac{L}{R}$



$\sigma_{rr} = 0$
$\sigma_{\theta\theta} = \frac{PR}{t}$
$\sigma_{zz} = \frac{PR}{2t}$

$\sigma_{VM} \rightarrow P_y = \frac{2\sigma_y t}{\sqrt{3}\,R}$

$K_{IC} = \frac{QPR}{t}\sqrt{\pi a}$

$\sigma_f < \sigma_y$ Fast Fracture

$\sigma_f = \sigma_y$ fast fracture $K_c = $ Fast Fracture Criteria

Microstructure Dev.

- S.S Å (0.1 nm)
- cluster / small precip ~nm
- phases (α, Fe, c) ~nm - μm
- grain ~mm - μm

Phase Diagrams [Locations, sizes of each phase, morphologies] → Kinetics of phase transformation

Fracture Criterion

$K_I = Q\sigma^\infty\sqrt{\pi a}$ $[Pa\sqrt{m}]$

$K_c = \sqrt{\frac{\pi}{4}E\Gamma_s}$ $K_I = K_c$ ← fast fracture

$\sigma_{22}(\text{local}) = \sigma_i$

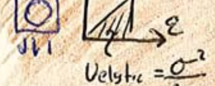
$2\sqrt{\frac{a}{\rho}}\,\sigma^\infty = \sqrt{\frac{E\Gamma_s}{a_0}}$

$\rho \sim a_0$ order mag

stress Derivation $\boxed{\sigma^\infty\sqrt{\pi a} = \sqrt{\frac{\pi E\Gamma_s}{4}}}$

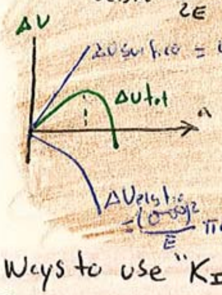
Energy derived

done by assuming energy changes. loss in energy storage due to the crack.



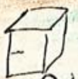
$\Delta U_{surf} = 2\Gamma_s(2at)$

$U_{elastic} = \frac{\sigma^2}{2E}$ $\Delta U_{elastic} = \left[\frac{(\sigma\cdot\sigma)^2}{2E}\right] t \cdot \pi a^2$



Griffith Energy

$\boxed{\sigma^\infty\sqrt{\pi a} = \sqrt{2\Gamma_s E}}$



$r_p = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_Y}\right)^2$

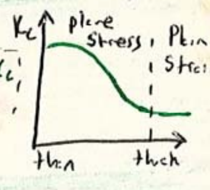
fail by fracture

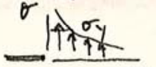
Ways to use "$K_I = K_c$" ; $r_p < \frac{1}{15}(a, w, w-a, l)$ min. $a\ w,\ w-a)$ $r_p <<$

① $K_I = Q\sigma^\infty\sqrt{\pi a}$

② Find crack length σ^∞, Q, K_c

③ Find fracture toughness of material Kc





plain strain fracture toughness $K_{IC} = Q\sigma^\infty\sqrt{\pi a}$

Kineticks of transformation

↑ cooling rate grain size ↓

↑ Tag − Tcool grain size ↓

Hall Patch $\tau_y \propto \beta d^{-1/2}$

Microstructure

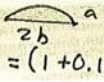
$P_{frac} = \frac{K_{IC}A_{\infty}}{Q(a)\sqrt{\pi a}}$

- Strengthening Mechanism
- G. B. Strengthening grain size ↓, τ_y ↑
- polycrystalline Material $\tau_{gross} = 1.5\,\tau_y$
- tensile yield stress $\sigma_y = 2 \times \tau_{gross} = 3\tau_y$

Elastic Fracture Mechanism

$\sigma_i = \sqrt{\frac{E\Gamma}{a}}$; $\Gamma\left(\frac{Ea_0}{100} - \frac{Ea_0}{10}\right)$, $\sigma_i \sim \frac{E}{10} \sim \frac{E}{3}$

elliptical crack $K_{IA} = Q_A\sigma^\infty\sqrt{\pi a}$

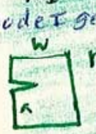


$Q_A = \left(1 + 0.12\left(1 - \frac{a}{b}\right)\right)\left(1 - .619\left(\frac{a}{b}\right)\right)^{1/2}$; $Q_B = \left(\frac{a}{b}\right)Q_A$

Lab 4 Notes

$K_I = Q\sigma^\infty\sqrt{\pi a}$ Fails $K_I \geq K_{IC}$

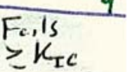
mode I geom under stress crack



$r_{IC} = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_Y}\right)^2$ $\sigma_Q = \frac{P_Q}{A}$ B.W

$K_Q = Q\sigma_Q\sqrt{\pi a}$

3Pt Bending



$P_{fracture} = \frac{4K_{IC}Bb^2}{6Q_{3PB}\sqrt{\pi a}\,S}$

Failure By Collapse $\sigma \rightarrow \sigma_y$ $M \rightarrow M_{LL}$

$M_L = \frac{\sigma_y Bb^2}{4}$

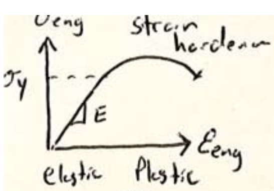
$P_{collapse} = \frac{M_{LL}4}{S} = \frac{\sigma_y Bb^2}{S}$

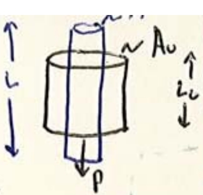
$M_{LL} = \frac{\sigma_y B(b-a)^2}{4}$ fails @ the lowest

$P_{collapse} = \frac{\sigma_y Bb^2}{S}\left(1 - \frac{a}{b}\right)^2$

Test

Small Scale





$\sigma_{eng} = P/A_0$ $\varepsilon_{eng} = \frac{\delta}{L_0}$

$\sigma_{true} = P/A$ $\varepsilon_{true} = Ln\left(\frac{L}{L_0}\right)$

Ductile
$q\uparrow$
Brittle
$q\downarrow$

$q = \frac{A_0 - A_f}{A_0}$

Tensor → Principal

$\sigma_{1'1'} = \sigma_{11}\cos^2\theta + \sigma_{22}\sin^2\theta + 2\sigma_{12}\sin\theta\cos\theta$

$\sigma_{2'2'} = \sigma_{11}\sin^2\theta + \sigma_{22}\cos^2\theta - 2\sigma_{12}\cos\theta\sin\theta$

$\sigma_{1'2'} = (\sigma_{22} - \sigma_{11})\cos\theta\sin\theta + \sigma_{21}(\cos^2\theta - \sin^2\theta$

$\sigma_{12}^{P} = \frac{1}{2}(\sigma_{11} + \sigma_{22}) \pm \sqrt{\left(\frac{\sigma_{11}-\sigma_{22}}{2}\right)^2 + \sigma_{12}^{\ 2}}$

$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{2\sigma_{12}}{\sigma_{11} - \sigma_{22}}\right)$

$\sigma_{mean} = \frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33}$

p = Pressure $= -\frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33}$

$\varepsilon_{true} = \ln(1 + \varepsilon_{eng})$

$\sigma_{true} = \sigma_{eng}(1 + \varepsilon_{eng})$

Pressure Stress $= \begin{bmatrix} p & & \\ & p & \\ & & p \end{bmatrix}$

$G = \frac{E}{2(1+\nu)}$

Poisson $\nu = \frac{-\varepsilon_{lateral}}{\varepsilon_{axial}}$

Deviateric Stress $= \bar{\bar{D}} = \bar{\bar{\sigma}} - \bar{\bar{p}}$

Stress Tensor

$$[\sigma] = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix}$$

Stress deviator

$\sigma' = [\sigma] - \sigma_M[1]$

$$\begin{bmatrix} \sigma_{11}-\sigma_M & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22}-\sigma_m & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33}-\sigma_m \end{bmatrix}$$

$\sum diag = 0$
no pressure in stress deviator

σ_m independent of Ref Frame

$\theta_p = 45°$ max normal stress

Rubber
Crosslinking density ↑
Increases Young modulus

Stress Equilibrium

$\frac{\partial\sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_{r\theta}}{\partial\theta} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0$

$\frac{\partial\sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta} + \frac{2\sigma_{r\theta}}{r} = 0$

$\sigma_{11} = \sigma_{rr}\cos^2\theta + \sigma_{\theta\theta}\sin^2\theta - \sigma_{r\theta}\sin 2\theta$

$\sigma_{22} = \sigma_{rr}\sin^2\theta + \sigma_{\theta\theta}\cos^2\theta + \sigma_{r\theta}\sin 2\theta$

Torsion Hooke's Law

$\tau = G\gamma$; $\gamma = (\rho/c)\gamma_{max}$ ρ radial position

$\tau_{max} = \frac{Tc}{J}$ J polar moment
T inted torge
c outer radius

$J = \frac{\pi}{2}c^4$ $J = \int_A r^2 dA$

$\tau = G\gamma$ τ_{max} = maximum shear stress

$\sigma_{\theta z} = G\varepsilon_{\theta z}$ $\sigma_{\theta z} = Gr\frac{d\alpha}{dz}$

$M = \int \sigma_{\theta z} r\, dA$

Bulk modulus $K = \frac{E}{3(1-2\nu)}$

$K = \frac{P}{\Delta}$ $\Delta = \frac{\Delta V}{V}$

$\varepsilon_{\theta z} = r\frac{d\alpha}{dz}$

Strain ~~tensor~~

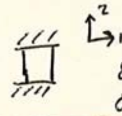
~~$\varepsilon_{11} = \frac{1}{2}\gamma_{11}$~~ $\gamma_{12} = \frac{1}{G}\sigma_{12}$ $\varepsilon_{12} = \frac{1}{2G}\sigma_{12}$

~~$\varepsilon_{22} = \frac{1}{2}\gamma_{22}$~~ $\gamma_{13} = \frac{1}{G}\sigma_{13}$ $\varepsilon_{13} = \frac{1}{2G}\sigma_{13}$

$\gamma_{23} = \frac{1}{G}\sigma_{23}$ $\varepsilon_{23} = \frac{1}{2G}\sigma_{23}$

Stress-Strain Temp Rel

$\varepsilon_{11} = \frac{1}{E}[\sigma_{11} - \nu(\sigma_{22} + \sigma_{33})] + \alpha\Delta T$



Compatibility
$\varepsilon_{22} = 0$ Continuity
$\sigma_{11} = 0$

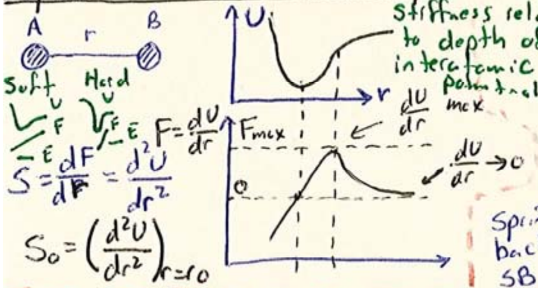
Yield Criterion

- Uniaxial $\sigma > \sigma_y$
- Multiaxial Von Mises $\bar{\sigma}$

$\sigma_{VM} = \left\{\frac{1}{2}\left[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{11} - \sigma_{33})^2 + (\sigma_{22} - \sigma_{33})^2\right] + 3(\sigma_{12}^{\ 2} + \sigma_{13}^{\ 2} + \sigma_{23}^{\ 2})\right\}^{1/2}$

Safety factor η ∴ $\frac{\sigma_{VM}}{\eta} = \sigma_{yield}$

Physical Basis of Elasticity Bond



stiffness related to depth of interatomic potential

$F = \frac{dU}{dr}$

$S = \frac{dF}{dr} = \frac{d^2U}{dr^2}$

$S_0 = \left(\frac{d^2U}{dr^2}\right)_{r=r_0}$

Residual Stress Unloading

$\left(\frac{1}{\rho}\right)_p = \left(\frac{1}{\rho}\right)_{loaded} - \underbrace{\left(\frac{1}{\rho}\right)_e}_{\text{Springback}}$

$\left(\frac{1}{\rho}\right)_e = \frac{M_{LL}}{EI} = \frac{\sigma_y bh^2}{E(bh^3/12)}$

Spring back SB. $\left(\frac{1}{\rho}\right)_e = \frac{\sigma_y}{E}\left(\frac{3}{h}\right)$

$h\downarrow$ SB ↑
$\frac{\sigma_y}{E}\uparrow$ SB ↑

Concepts

$\tau_{ideal} \gg \sigma_y$ Differ by 30 of M dislocations.

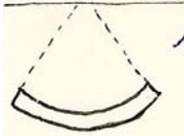
For glass ceramic $\tau_{ideal} \sim \sigma_y$

σ_y 0.2% in strain shift

E is from interatomic bonding strength

$\frac{\Delta V}{V} \sim \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$ Find $\frac{\Delta V}{V} = 0$ $\nu = 0.5$

Elastic Plastic Beam Bending



ρ = radius of curvature
$K = 1/\rho$

$\varepsilon_{11} = \frac{l - l_0}{l_0}$

$= \frac{(\rho - x_2)d\theta - \rho d\theta}{\rho d\theta}$

$\varepsilon_{11} = \frac{-x_2}{\rho}$

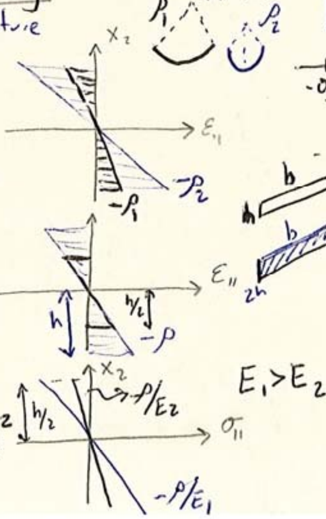
$\sigma_{11} = E\varepsilon_{11}$

$\sigma^{surface} = \sigma_y$

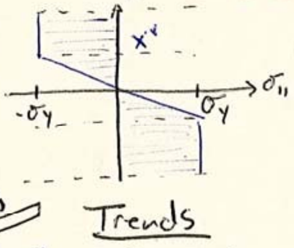
$\boxed{\frac{EI}{\rho} = M}$ $\to \frac{E}{\rho} = \frac{M}{I}$

$\square \Rightarrow I = \frac{1}{2}bh^3$

$M_e = \frac{\sigma_y bh^2}{6}$ $M_{LL} = \frac{\sigma_y bh^2}{4}$



$E_1 > E_2$



Trends

$E\uparrow$ $\Delta K\downarrow$
$h\uparrow$ $\Delta K\downarrow$
$\sigma_y\downarrow$ $K_e\downarrow$

$M_{LL} = 1.5\, M_e$

$M = \int -\sigma_{11} x_2\, dA$ $dA = b\,dx_2$

$= 2\int_{x_2^*}^{h/2} \sigma_y x_2\, dA + \int_{-x_2^*}^{x_2} -E\left(-\frac{x_2}{\rho}\right)x_2\, dA$

$x_2^* = \frac{\sigma_y \rho}{E}$

$M = \sigma_y b\left[\frac{h^2}{4} - \frac{1}{3}x_2^{*\,2}\right.$

$\Delta K = \frac{M_{load}}{EI}$

$K_{loaded} - K_{unloaded} = \Delta K$

Stiff Circular Plate

$\delta = \frac{3}{4\pi}\frac{wa^2}{Et^3}(1-\nu^2)\left(\frac{3+\nu}{1+\nu}\right)$ $w = S\delta$

Shear vs. Yield

$\sigma = F/A$ yield $\tau_y = \frac{\sigma_y}{2} > \frac{1}{2}$ Shear

$\tau_{r\theta} = F/SA$ $P_{in} = P_{out}$

$\dot{u}_I$ // $\angle\theta$ $\vec{u}_{II}$ $P \cdot \dot{u}_I = \frac{bl}{\cos\theta}\dot{u}_{II}\tau$

$\sigma_y = \sigma_{VM} = \sqrt{3}\tau_y$ $\dot{u}_I = \dot{u}_{II}\sin\theta$

Bonds		S_o (N/m)	E (GPa)	Melting Pt
Covalent	C-C	50-180	200-1000	3500°C
Metallic	Cu-Cu	15-75	60-300	1000°C
Ionic	Na-Cl	8-24	32-96	800 C
Vander walls	Polymers	0.5-1	2-4	

Th m

$I = I_o + Ar^2$

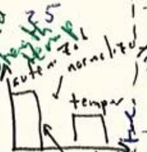
$I = \int y^2 dA$

Shear Strength	E (GPa)	$G \approx 3/8 E$	$\tau_i = \frac{G}{2\pi}$ GR	τ_{exp} (GPa)
Al	20	25	4	1×10^{-3}
Fe	200	75	12	0.4
Diamond	1300	500	80	

Materials	E (GPa)	ρ (g/cm³)
Steel	200	7.9
Al	70	2.7
polycarbonate	2-3	1.2
Composite	150	1.5
Ti	120	4.5

	Quenched	Quenched & Tem	Normalized	Annealed
Energy J	6		29	36
MPa HRA	1829		647	537
MPa Yield Str	1607		1067	647

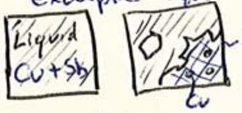
25



quenched & tempered stronger with temper

material	Density Mg/m³	Modulus GPa	$E^{1/2}/\rho$ M_1	ϕ_B^e	$M_2 \left(\frac{(\phi_B^e E)^{1/2}}{\rho}\right)$
steel	7.85	205	1.8	20	8.2
Al	2.7	70	3.1	15	12
GFRP	1.75	28	2.9	8	8.5
wood	.99	13.5	4.1	2	5.8
Bambo	.6-.8	16-18	6	2.2	9

example type



Eutectoid

pearlite, formed Since it allows for faster C diffusion the Θ

Starts at edges

Slow Cooling

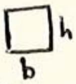
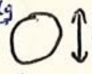
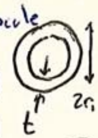
Thinner faster diffusion

Fe_3C precipitates limited by diffusion rate out of the lattice.

$t \uparrow \; d \uparrow$

Fast Cooling

martensite quenching

	Area	I, ϕ_B^e	Φ_T^e	ϕ_B^f	ϕ_T^f
 h b	bh	$\frac{bh^3}{12}$, $\frac{h}{b}$	$2.38\frac{h}{b}(1-.58\frac{b}{h})$	$\left(\frac{h}{b}\right)^{1/2}$	$1.6\sqrt{\frac{b}{h}}\frac{1}{(1+.6\frac{b}{h})}$
 2r	πr^2	$\frac{\pi}{64}d^4$, ~~1.14~~ $\frac{3}{\pi}$	~~$\frac{2}{2\sqrt{\pi}}$~~ 1.14	~~1.35~~ $\frac{3}{2\sqrt{\pi}}$	~~$\frac{4}{3\sqrt{\pi}}$~~ 1.35
 $2r_o$ $2r_i$ t	$\pi(r_o^2-r_i^2)$	, $\frac{3}{\pi}\frac{r}{t}$	$1.14\left(\frac{r}{t}\right)$	$\frac{3}{\sqrt{2\pi}}\sqrt{\frac{r}{t}}$	$1.91\sqrt{r/t}$

Solidification

$$\nu \approx \frac{d}{6h} e^{-q/kt} \qquad \frac{\Delta H(T_m - T)}{T_m}$$

Tmelt

h = Planks Constant $6.62 \times 10^{-34} \frac{m^2 kg}{s}$

q = energy barrier

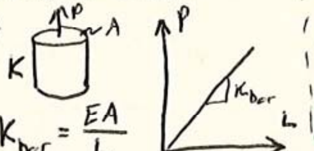
K = Boltzmann Const = 1.4×10^{-23} J/molecule

ΔH = Latent heat of fusion

ν = Solidification rate molecules per second per molecule

Materials Selection

1)



$$K_{bar} = \frac{EA}{L}$$

Bending Stiffness $P = K_{BB}\delta$ Bending Stiffness

$$K_{BB} = \frac{C_i E I}{L^3}$$

$C_i = 3$ Cantilever

$C_i = 48$ Central load with 2pt Support.

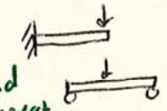


Plate Bending Stiffness.

$$K_{PB} = \frac{C_2 E \omega t^3}{12 L^3}$$

$$\phi_B^e = \frac{I}{I_o}$$

Squared ref

Axial Loading

$F = \sigma_y A$

$A =$

$$\sigma = \frac{M y_m}{I}$$

$$z = \frac{I}{y_m} \quad \phi_B^f = \frac{z}{z_o} = \frac{6z}{A^{3/2}}$$

NA

grains size small

Quenched normalized high Yield

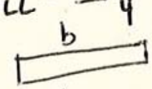
Hardness ~ ultimate tensile strength ~ Yield strength

Ceramics | CFRP

Brittle | Cost

Bending

$$M_{LL} = \frac{\sigma_y b h^2}{4}$$

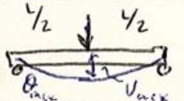
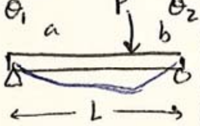
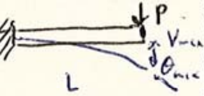
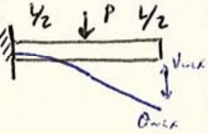
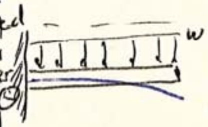
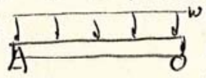


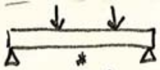
$h =$

Cantilevers / 3pt Bending etc Loading

821

⑥

Beam	slope	Deflection	Elastic curve
 L/2 L/2 θ_{max} V_{max}	$\theta_{max} = \frac{-PL^3}{16EI}$	$V_{max} = \frac{-PL^3}{48EI}$	$V = \frac{-Px}{48EI} \cdot (3L^2 - 4x^2)$
 θ_1 a P b θ_2 L	$\theta_1 = \frac{-Pab(L+b)}{6EIL}$ $\theta_2 = \frac{Pab(L+a)}{6EIL}$	$= \frac{-Pba}{6EIL} \cdot () (L^2 - b^2 - a^2)$	$V = \frac{-Pbx}{6EIL} \cdot (L^2 - b^2 - x^2)$
 P V_{max} θ_{max} L	$\theta_{max} = \frac{-PL^2}{2EI}$	$V_{max} = \frac{-PL^3}{3EI}$	$V = \frac{-Px^2}{6EI}(3L-x$
 L/2 P L/2 V_{max} θ_{max}	$= \frac{-PL^2}{8EI}$	$V_{max} = \frac{-5PL^3}{48EI}$	$V = \frac{-Px^2}{6EI}\left(\frac{3}{2}L-\right.$ $0 \le x \le L/2$ $V = \frac{-PL^2}{24EI}(3x - \frac{L}{2}$ $L/2 \le x \le L$
 w	$\theta_{max} = \frac{-wL^3}{6EI}$	$V_{max} = \frac{-wL^4}{8EI}$	$V = \frac{-wx^2}{24EI} \cdot () (x^2 - 4Lx - 6L$
 w	$\theta_{max} = \frac{-wL^3}{24EI}$	$V_{max} = \frac{-5wL^4}{384EI}$	$V = \frac{-wx}{24EI} \cdot () (x^3 - 2Lx^2 + L^3$



$M^* = PL/8$

$$M = 2\int_0^{h/2} \sigma_y\, y b\, dy$$

Relate σ_y to σ_{nom} plug into Fracture criteria

Lab 5 $m = \rho A L$ minimize mass

$S_B = C_1 \frac{EI}{L^3}$ δ Bending $M_1 = \frac{E^{1/2}}{\rho}$

Tube Bending Varying shape

$$\phi_b^e = \frac{S}{S_o} = \frac{12 I}{A^2} \qquad M_2 = \frac{((\phi_b^e)E}{\rho}$$

Tube Bending: Resistance failure

$$\sigma = \frac{M y_m}{I} < \sigma_f \qquad M_3 = \left[\frac{\sigma_f^{2/3}}{\rho}\right]$$

Torsion

$$S_T = \frac{KG}{L}; \; \phi_T^e = \frac{K}{K_o} = \frac{S_T}{S_o}$$

$$\phi_T^e = 7.14\frac{K}{A^2} \qquad S_T = \frac{GA^2\phi_T^e}{7.14 L}$$

$m = \rho A L$

$$M_4 = \frac{\phi_T^e G}{\rho^{1/2}} \qquad G \approx 3/8 E$$

Cost

$$C = mC_m = \rho A L C_m$$

$$M_5 = \frac{E^{1/2}}{\rho C_m}$$